CHAPTER 27 TORQUE

EXERCISE 122, Page 271

1. Determine the torque developed when a force of 200 N is applied tangentially to a spanner at a distance of 350 mm from the centre of the nut.

Torque T = Fd, where force F = 200 N and distance, d = 350 mm = 0.35 m

Hence, **torque, T** = (200)(0.35) = **70 N m**

2. During a machining test on a lathe, the tangential force on the tool is 150 N. If the torque on the lathe spindle is 12 N m, determine the diameter of the work-piece.

Torque T = Fr, where torque T = 12 N m, force F = 150 N at radius r

Hence, 12 = (150)(r)

from which, radius, $r = \frac{12}{150} = 0.08$ m = 80 mm

Hence, **diameter** = 2 × 80 = **160 mm**

EXERCISE 123, Page 273

1. A constant force of 4 kN is applied tangentially to the rim of a pulley wheel of diameter 1.8 m attached to a shaft. Determine the work done, in joules, in 15 revolutions of the pulley wheel.

Torque T = Fr, where F = 4000 N and radius r = $\frac{1.8}{2}$ = 0.9 m

Hence, torque T = (4000)(0.9) = 3600 N m

Work done = Tθ joules, where torque, T = 3600 N m and angular displacement,

θ = 15 revolutions = 15 × 2π rad = 30π rad.

Hence, **work done** = Tθ = (3600)(30π) = 339.3×10^3 = **339.3 kJ**

2. A motor connected to a shaft develops a torque of 3.5 kN m. Determine the number of revolutions made by the shaft if the work done is 11.52 MJ.

Work done = Tθ joules, where work done = 11.52×10^6 J and torque, T = 3500 N m

Hence, $11.52\times10^6 = 3500\times\theta$

from which, angular displacement, $\theta = \frac{11.52\times10^6}{3500}$ = 3291.43 rad

and **number of revolutions** = $\frac{3291.43}{2\pi}$ = **523.8 rev**

3. A wheel is turning with an angular velocity of 18 rad/s and develops a power of 810 W at this speed. Determine the torque developed by the wheel.

Power P = Tω, where P = 810 W and angular velocity, ω = 18 rad/s

Hence, 810 = T × 18

from which, **torque, T** = $\frac{810}{18}$ = **45 N m**

4. Calculate the torque provided at the shaft of an electric motor that develops an output power of 3.2 hp at 1800 rev/min.

Power, $P = 2\pi nT$, where power $P = 3.2$ h.p. $= 3.2 \times 745.7 = 2386.24$ W and $n = \dfrac{1800}{60} = 30$ rev/s

Hence, $2386.24 = 2\pi \times 30 \times T$

from which, **torque,** $\mathbf{T} = \dfrac{2386.24}{2\pi \times 30} = \mathbf{12.66\ N\ m}$

5. Determine the angular velocity of a shaft when the power available is 2.75 kW and the torque is 200 N m.

Power, $P = 2\pi nT$, where power $P = 2750$ W and torque $T = 200$ N m

Hence, $2750 = 2\pi \times n \times 200$

from which, $n = \dfrac{2750}{2\pi \times 200} = 2.1884$ rev/s

Angular velocity, $\omega = 2\pi n = 2\pi \times 2.1884 = \mathbf{13.75\ rad/s}$

6. The drive shaft of a ship supplies a torque of 400 kN m to its propeller at 400 rev/min. Determine the power delivered by the shaft.

Power, P $= \omega T = 2\pi nT = 2\pi \times \dfrac{400}{60} \times 400 \times 10^3$

$= 16.76 \times 10^6$ W $=$ **16.76 MW**

7. A motor is running at 1460 rev/min and produces a torque of 180 N m. Determine the average power developed by the motor.

Power, P = $\omega T = 2\pi nT = 2\pi \times \frac{1460}{60} \times 180 = 27.52 \times 10^3$ W = **27.52 kW**

8. A wheel is rotating at 1720 rev/min and develops a power of 600 W at this speed. Calculate (a) the torque, (b) the work done, in joules, in a quarter of an hour.

(a) Power, $P = 2\pi nT$ hence, $600 = 2\pi \times \frac{1720}{60} \times T$

from which, **torque, T** = $\frac{600 \times 60}{2\pi \times 1720}$ = **3.33 N m**

(b) Work done = $T\theta$, where torque T = 3.33 N m and

angular displacement in 15 minutes = (15×1720)rev = $(15 \times 1720 \times 2\pi)$ rad.

Hence, **work done** = $T\theta = (3.33)(15 \times 1720 \times 2\pi) = 540 \times 10^3$ J = **540 kJ**

EXERCISE 124, Page 275

1. A shaft system has a moment of inertia of 51.4 kg m^2. Determine the torque required to give it an angular acceleration of 5.3 rad/s^2.

Torque, T = Iα, where moment of inertia I = 51.4 kg m^2 and angular acceleration, α = 5.3 rad/s^2.

Hence, **torque, T** = Iα = (51.4)(5.3) = **272.4 N m**

2. A shaft has an angular acceleration of 20 rad/s^2 and produces an accelerating torque of 600 N m. Determine the moment of inertia of the shaft.

Torque, T = Iα, where torque T = 600 N m and angular acceleration, α = 20 rad/s^2.

Hence, $600 = I \times 20$

from which, **moment of inertia of the shaft, I** = $\frac{600}{20}$ = **30 kg m^2**

3. A uniform torque of 3.2 kN m is applied to a shaft while it turns through 25 revolutions. Assuming no frictional or other resistance's, calculate the increase in kinetic energy of the shaft (i.e. the work done). If the shaft is initially at rest and its moment of inertia is 24.5 kg m^2, determine its rotational speed, in rev/min, at the end of the 25 revolutions.

Work done = Tθ = 3200 $\times$ (25 $\times$ 2π) = **502.65 kJ**

Increase in kinetic energy = 502650 J = $I\left(\frac{\omega_2^2 - \omega_1^2}{2}\right)$ where I = 24.5 kg m^2 and $\omega_1 = 0$

Hence, $$502650 = 24.5\left(\frac{\omega_2^2 - 0}{2}\right)$$

from which, $$\omega_2^2 = \frac{502650 \times 2}{24.5} \quad \text{and} \quad \omega_2 = \sqrt{\frac{502650 \times 2}{24.5}} = 202.565 \text{ rad/s}$$

Hence, **rotational speed** = $202.565 \text{ rad/s} \times \dfrac{60 \text{ s/min}}{2\pi \text{ rad/rev}}$ = **1934 rev/min**

4. An accelerating torque of 30 N m is applied to a motor, while it turns through 10 revolutions. Determine the increase in kinetic energy. If the moment of inertia of the rotor is 15 kg m^2 and its speed at the beginning of the 10 revolutions is 1200 rev/min, determine its speed at the end.

Increase in kinetic energy = work done = $T\theta = 30 \times (10 \times 2\pi)$ = **1885 J** or **1.885 kJ**

Increase in kinetic energy = 1885 J = $I\left(\dfrac{\omega_2^2 - \omega_1^2}{2}\right)$

where I = 15 kg m^2 and $\omega_1 = 1200 \times \dfrac{2\pi}{60} = 40\pi = 125.664$ rad/s

Hence, $$1885 = 15\left(\frac{\omega_2^2 - 125.664^2}{2}\right)$$

from which, $$\omega_2^2 - 125.664^2 = \frac{1885 \times 2}{15} = 251.333$$

Hence, $$\omega_2^2 = 251.333 + 125.664^2 = 16042.774$$

and $$\omega_2 = \sqrt{16042.774} = 126.66 \text{ rad/s}$$

Hence, **final speed** = $126.66 \text{ rad/s} \times \dfrac{60 \text{ s/min}}{2\pi \text{ rad/rev}}$ = **1209.5 rev/min**

5. A shaft with its associated rotating parts has a moment of inertia of 48 kg m^2. Determine the uniform torque required to accelerate the shaft from rest to a speed of 1500 rev/min while it turns through 15 revolutions.

Work done = increase in kinetic energy = $T\theta = I\left(\dfrac{\omega_2^2 - \omega_1^2}{2}\right)$

where I = 48 kg m^2, $\omega_1 = 0$ and $\omega_2 = 1500 \times \dfrac{2\pi}{60} = 157.08$ rad/s

Hence, $T\theta = I\left(\frac{\omega_2^2 - \omega_1^2}{2}\right)$ i.e. $T(15 \times 2\pi) = 48 \times \left(\frac{157.08^2 - 0}{2}\right) = 592179$

from which, **torque, T** = $\frac{592179}{15\times 2\pi}$ = **6283 N m** or **6.283 kN m**

6. A small body, of mass 82 g, is fastened to a wheel and rotates in a circular path of 456 mm diameter. Calculate the increase in kinetic energy of the body when the speed of the wheel increases from 450 rev/min to 950 rev/min.

Increase in kinetic energy = $I\left(\frac{\omega_2^2 - \omega_1^2}{2}\right) = m\,r^2\left(\frac{\omega_2^2 - \omega_1^2}{2}\right)$

$$= (0.082)\left(\frac{0.456}{2}\right)^2\left(\frac{\left(\frac{950\times 2\pi}{60}\right)^2 - \left(\frac{450\times 2\pi}{60}\right)^2}{2}\right)$$

$$= (0.082)(0.051984)(\left(\frac{99.484^2 - 47.124^2}{2}\right)$$

= **16.36 J**

7. A system consists of three small masses rotating at the same speed about the same fixed axis. The masses and their radii of rotation are: 16 g at 256 mm, 23 g at 192 mm and 31 g at 176 mm. Determine (a) the moment of inertia of the system about the given axis, and (b) the kinetic energy in the system if the speed of rotation is 1250 rev/min.

(a) **Moment of inertia, I** = $\sum mr^2 = (0.016)(0.256)^2 + (0.023)(0.192)^2 + (0.031)(0.176)^2$

$$= 1.0486\times 10^{-3} + 8.4787\times 10^{-4} + 9.6026\times 10^{-4}$$

$$= \mathbf{2.857\times 10^{-3}\,kg\,m^2}$$

(b) **Kinetic energy in the system** = $I\frac{\omega^2}{2} = \left(2.857\times10^{-3}\right)\left(\frac{\left(\frac{1250\times2\pi}{60}\right)^2}{2}\right)$ = **24.48 J**

EXERCISE 125, Page 277

1. A motor has an efficiency of 72% when running at 2600 rev/min. If the output torque is 16 N m at this speed, determine the power supplied to the motor.

Power output, $P = 2\pi nT$

$= 2\pi\,(2600/60)(16) = 4356.34$

Efficiency $= \dfrac{\text{power output}}{\text{power input}} \times 100\%$ hence $72 = \dfrac{4356.34}{\text{power input}} \times 100$

from which, **power input** $= \dfrac{4356.34}{72} \times 100$ = **6050 W** or **6.05 kW**

2. The difference in tensions between the two sides of a belt round a driver pulley of radius 240 mm is 200 N. If the driver pulley wheel is on the shaft of an electric motor running at 700 rev/min and the power input to the motor is 5 kW, determine the efficiency of the motor. Determine also the diameter of the driven pulley wheel if its speed is to be 1200 rev/min.

Power output from motor $= (F_2 - F_1)r_x\,\omega_x$

$(F_2 - F_1) = 200$ N, radius $r_x = 240$ mm $= 0.24$ m and angular velocity, $\omega_x = \dfrac{700 \times 2\pi}{60}$ rad/s

Hence, power output from motor $= (F_2 - F_1)r_x\,\omega_x = (200)(0.24)\left(\dfrac{700 \times 2\pi}{60}\right) = 3518.58$ W

Power input = 5000 W

Hence, **efficiency of the motor** $= \dfrac{\text{power output}}{\text{power input}} = \dfrac{3518.58}{5000} \times 100 =$ **70.37%**

$\dfrac{r_x}{r_y} = \dfrac{n_y}{n_x}$ from which, driven pulley wheel radius, $r_y = \dfrac{n_x r_x}{n_y} = \dfrac{700 \times 0.24}{1200} = 0.14$ m

from which, **diameter of driven pulley wheel** = 2 × radius = 2 × 0.14 = **0.28 m** or **280 mm**

3. A winch is driven by a 4 kW electric motor and is lifting a load of 400 kg to a height of 5.0 m. If the lifting operation takes 8.6 s, calculate the overall efficiency of the winch and motor.

The increase in potential energy is the work done and is given by mgh (see Chapter 20), where mass, m = 400 kg, g = 9.81 m/s^2 and height h = 5.0 m.

Hence, work done = mgh = (400)(9.81)(5.0) = 19.62 kJ.

Input power = 4 kW = 4000 W Output power = $\frac{\text{work done}}{\text{time taken}} = \frac{19620}{8.6} = 2281.4$ W

Efficiency = $\frac{\text{output power}}{\text{input power}} \times 100 = \frac{2281.4}{4000} \times 100 =$ **57.03%**

4. A belt and pulley system transmits a power of 5 kW from a driver to a driven shaft. The driver pulley wheel has a diameter of 200 mm and rotates at 600 rev/min. The diameter of the driven wheel is 400 mm. Determine the speed of the driven pulley and the tension in the slack side of the belt when the tension in the tight side of the belt is 1.2 kN.

r_x = 100 mm = 0.1 m, n_x = 600 rev/min, r_y = 200 mm = 0.2 m

$\frac{r_x}{r_y} = \frac{n_y}{n_x}$ from which, **speed of driven pulley**, $n_y = \frac{r_x n_x}{r_y} = \frac{0.1 \times 600}{0.2}$ = **300 rev/min**

Available power = $(F_2 - F_1) r_x \omega_x$

i.e. $5000 = (1200 - F_1)(0.1)\left(600 \times \frac{2\pi}{60}\right)$

i.e. $(1200 - F_1) = \frac{5000}{0.1\left(600 \times \frac{2\pi}{60}\right)} = 795.8$

Hence, **tension in slack side of belt**, $\mathbf{F_1}$ = 1200 – 795.8 = **404.2 N**

5. The average force on the cutting tool of a lathe is 750 N and the cutting speed is 400 mm/s. Determine the power input to the motor driving the lathe if the overall efficiency is 55%.

Force resisting motion = 750 N and velocity = 400 mm/s = 0.4 m/s

Output power from motor = resistive force $\times$ velocity of lathe (from Chapter 20)

$$= 750 \times 0.4 = 300 \text{ W}$$

$$\text{Efficiency} = \frac{\text{power output}}{\text{power input}} \times 100$$

hence $$55 = \frac{300}{\text{power input}} \times 100$$

from which, **power input** = $300 \times \frac{100}{55}$ = **545.5 W**

6. A ship's anchor has a mass of 5 t. Determine the work done in raising the anchor from a depth of 100 m. If the hauling gear is driven by a motor whose output is 80 kW and the efficiency of the haulage is 75%, determine how long the lifting operation takes.

The increase in potential energy is the work done and is given by mgh (see Chapter 20), where mass, m = 5 t = 5000 kg, g = 9.81 m/s^2 and height h = 100 m

Hence, **work done** = mgh = (5000)(9.81)(100) = **4.905 MJ**

Input power = 80 kW = 80000 W

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} \times 100$$

hence $$75 = \frac{\text{output power}}{80000} \times 100$$

from which, output power = $\frac{75}{100} \times 80000 = 60000 \text{ W} = \frac{\text{work done}}{\text{time taken}}$

Thus, **time taken for lifting operation** = $\frac{\text{work done}}{\text{output power}} = \frac{4.905 \times 10^6 \text{ J}}{60000 \text{ W}}$

= **81.75 s** = **1 min 22 s** to the nearest second.

EXERCISE 126, Page 278

Answers found from within the text of the chapter, pages 270 to 277.

EXERCISE 127, Page 278

1. (d) **2.** (b) **3.** (c) **4.** (a) **5.** (c) **6.** (d) **7.** (a) **8.** (b) **9.** (c) **10.** (d) **11.** (a) **12.** (c)
